INFLUENCE OF LIMITS SET TO LATERAL DISPLACEMENT ON THE DETERMINATION OF MEAN MULTIPLE COULOMB SCATTERING IN NUCLEAR EMULSIONS

PREM K ADITYA

INDIAN INSTITUTE OF TECHNOLOGY
NEW DELHI-29 (INDIA)

(*Received July 27, 1965, Resubmitted December 6, 1965*)

ABSTRACT Multiple scattering constant has been determined in nuclear emulsions using 8 GeV μ-mesons. The influence of limits set on lateral scattering of tracks has been investigated It has been shown that an underestimation to an extent of upto 25% may be introduced by restricting track displacements to say about 50 μm in 10 cm travel Such limits are generally imposed upon by the requirement of having close pairs of tracks to avoid spurious scattering in relative scattering measurements or due to the limited width of the field of view of the microscopes and due to finite thickness of the emulsion layer

INTRODUCTION

In recent years there have been many investigations on multiple Coulomb scattering, particularly the determination of the scattering constant in nuclear emulsions. The scattering constant for a singly charged particle is defined by the relation, (Voyvodic and Pickup, 1952) ·

$$\alpha = \frac{KZt^{\frac{1}{2}}}{p\beta c}$$

where α is the mean scattering angle in degrees, $p\beta c$ is momentum times velocity in MeV and t is the cell length in 100μm units The scattering constant, K in MeV deg. (100μm)$^{\frac{1}{2}}$ is known to be about 25 to 30. The determination of K involves measurement of the mean angle of scattering, the method generally adopted is the coordinate method developed at Bristol (Fowler 1950).

In an investigation carried out recently (Aditya 1964a, 1964b) multiple scattering measurements had been made on tracks of 8 GeV μ-mesons. It has been shown that because μ-mesons have only electromagnetic interaction, in contrast with protons and π-mesons which have also a finite cross section for inelastic interaction, the multiple scattering is more precisely defined for them.

STATEMENT OF THE PROBLEM

From the observed multiple scattering for particles of known energy the scattering constant K should be directly determinable.

Ideally the observed scattering should be entirely genuine. There should be no additional component and there should be no restrictions imposed upon by the experimental method. In actual practice, however, the emulsion suffers from the presence of spurious scattering (see, for example, Biswas *et al*, 1955, Aditya *et al* 1963) which severely distorts the frequency distribution of angular scatters and leads to an over-estimation of scattering.

In order to eliminate spurious scattering one of the generally accepted methods has been the relative scattering of "close tracks". It has been shown (Biswas *et al* 1957) that, spurious scattering increases with increase of mutual separation between two tracks of a pair Limits are thus generally set to track separation depending upon the magnitude of spurious scattering in the plates. Limits are set sometimes also in order to avoid the inconvenience of realigning a track moving out of the field of view due to large angle scattering and curvature due to genuine scattering. Such a limit is sometimes enforced upon the observer by the finite thickness of emulsion. This phenomena called flat-chamber effect (Menon *et al*. 1951) is severe for low energy particles restricted by limits set on lateral displacement in the vertical plane

It has been shown (Aditya, 1964a) that the r.m.s. value of displacement y_{rms} (μm) of a track of energy $p\beta c$ (GeV) with repect to original direction in traversing a distance $x(cm)$ in emulsion is given as :

$$y_{rms} = \frac{50\ x^{3/2}}{p\beta c}$$

In deriving this relation it has been assumed that the angular scatters have a normal distribution. Thereby this relation gives the fraction of particles which

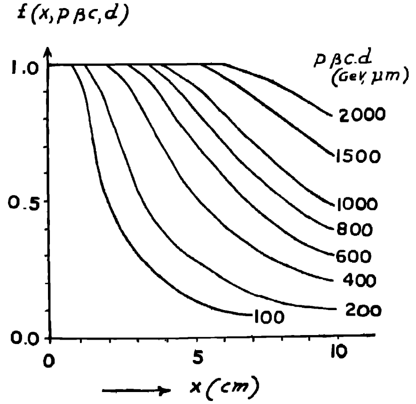

Fig. 1. The probability, $f(x, p\beta c, d)$ that the track of a particle of energy $p\beta c$(GeV) will have a displacement less than $d(\mu m)$ in traversing a distance $x(cm)$ in emulsion, plotted against x, for various values of the product $p\beta c.d$ ($GeV\ \mu m$), letters attached to the curves.

undergoing genuine scattering will confine themselves to the prescribed limits of lateral displacement. By making use of standard tables for area under the curve of the Gaussian distribution, the fraction of particles that will confine their lateral displacement to several other limits, measured in the units of the standard deviation, has been calculated The results are plotted in Fig 1 The ordinate gives here $f(x, p\beta c, d)$, the fraction of tracks for which a particle of energy $p\beta c$ (GeV) is expected to have a displacement less than $d(\mu_m)$ in traversing a distance $x(cm)$ in emulsion. For 5 cm traversal and $f(x, p\beta c. d) = 0.5$, the product $p\beta c.d$ is seen to be 400 GeV.μm. Thus only 50% of particles of energy 10GeV, undergoing genuine scattering, are expected to have a lateral displacement within 40μm, measured with respect to the initial direction at $x = 0$. A small value of f indicates that most of the particles (i e , fraction $1-f$) will not have their tracks limited to the prescribed lateral displacement If measurements are made only on such tracks as remain within limits, the observed scattering would be underestimated

The curves in Fig.1, can be converted into $f(x, p\beta c, s)$ for mutual separation $s(\mu m)$ between a pair of tracks of equal energy $p\beta c$(GeV), starting initially with almost no angle between them In such a case the probability that the angle will only increase due to scattering is $\frac{4\pi-\Omega}{4\pi} \sim 1$, where Ω is the solid angle defined by the aperture of the pair The average separation at small distances may be assumed to be $\sqrt{2}$ times the individual displacement, (Aditya 1959). Referring to Fig. 1, and the case $x = 5$ cm, $f(x, p\beta c, d) = 0.5$ as considered above, the product $p\beta c.s$ would be $400/\sqrt{2}$. Thus for pairs of tracks of 10GeV particles transversing 5 cm, only half of the total number of pairs will have a separation less than about 30μm or that only about 35% will have separation to within 50 μm.

In the present work we have determined the underestimation in scattering, due to such restrictions imposed on lateral displacement.

EXPERIMENTAL MATERIAL

Multiple scattering measurements on tracks of 8 GeV μ-mesons made for the earlier work (Aditya 1964a) have been employed. The momentum of the μ-mesons was precisely known in each region Suitably selected tracks were followed for over 10 cm length.

The overall noise, (personal cum that of the equipment), had been measured from time to time using standard techniques (Biswas *et al.*, 1955); it varied from 0.1μm to 0.15μm for different tracks. The observations had been restricted to regions having no measurable distortion, studied in connection with another work (Aditya and Puri 1964) where we have shown that spurious scattering can be understood entirely in terms of distortion Spurious scattering is estimated to be at most of the same order as the overall noise

Particular attention had been paid to bias, in the selection of tracks, arising due to limited lateral displacement. From equation (1) above and making allowance for two standard deviations it was known that 95% of particles would restrict themselves to prescribed lateral displacement. On this basis only such tracks were picked up for measurement as were expected to stay within the thickness of emulsion for entire path length in the plate. No limits were set on the displacement in the projected plane. These conditions ensured a bias-free sample of tracks.

ANALYSIS

The influence of the setting up of limits has been experimentally determined by calculating mean scattering for different arbitrary limits set to the lateral displacement of μ-meson tracks with respect to initial direction. Limits have been set at values from 10μm to 150μm over 5 cm traversal of emulsion, the results on scattering constant for 5 cm cell length are given in table I. The percentage under-estimation is given in column 3, K_{true} having been taken as 31 0 (Voyvodic and Pickup 1952). The influence of setting arbitrary limits to lateral displacement is seen to be considerable. The data allow an empirical relation to be written as :

$$\text{percentage underestimation} = 30(1-f_y)$$

where f_y, is the value of the function $f(x, p\beta c, d)$ for displacement in the projected plane. This empirical relation is expected to hold upto about 25% underestimation only.

TABLE I

Dependence of K on Limits Set to Lateral Scattering for 8 GeV μ-mesons at 5mm Cell Length

Maximum lateral displacement (μm)	K observed [Mev deg. $(100\ \mu)^{1/2}$]	Underestimation (percentage)	$f(x, p\beta c, d)$
10	24 0±1 8	23±6	0 10
25	25 6±1 9	17±6	0 28
50	26.4±2.0	15±6	0 52
100	28 7±2.1	7±6	0.83
150	30 2±2 2	2±6	0 95

The observations in depth were not taken at each successive cell but only at few points along the tracks. However an approximate estimate of the influence of setting up of limits in the depth direction has been similarly made ; we find empirically:

$$\text{percentage underestimation} = 8(1-f_z)$$

These considerations may now be applied to data available in literature where limits had been set to track separation. The underestimation in K due to restrictions on lateral scattering has been calculated by using emperical relations given above results are given in table II. It is seen that K may be underestimated by about 5% to 20% by this type of restricted sampling. The bias for obvious reasons is large in experiments involving larger length per track. It is quite possible that the apparent decrease of scattering constant with cell length (Hossain *et al.* 1961, Pal *et al.* 1963) is partly associated with this effect.

TABLE II

Underestimation of K calculated for limits set to lateral displacement by other investigators

Reference	Particle	Energy (GeV)	Limits d or s(μm)	x(cm)	Under-estimation (percentage)
Brisbout *et al.* (1956)	π	4 5	sy = 50 sz = 20	1 5	0.6 7.2
Biswas *et al.* (1957)	p	6 2	sy = 200 sz = 50	5 5	3.6 5.6
Aditya *et al.* (1963)	p	27	sy = 100 sz = 50	8 8	3.0 3.2
Kamal *et al.* (1964)	p	24	sy = 50 sz = 20	10 10	17.4 6 7
Hossain *et al.* (1961a)	π	16 2	dy = 50	10	18
Bozoki *et al* (1964)	π	17.2	dy = 50	10	17.4

The influence of this bias is not felt in the scattering constant values of all the work referred to in Table II, presumably due to different level of spurious scattering. If the level of spurious scattering is low, then the scattering on the whole is underestimated, as in the case of, Brisbout *et al.* (1956) and Hossain *et al.* 1961), leading to underestimation of K. However, if spurious scattering in the plates is large, as is for other references in Table II, then in spite of best effort it may not be possible to eliminate entirely its influence. The amount of spuriouss scattering that continues to remain in the result would over-estimate the scattering It would in the first instance compensate for the underestimation due to bias and eventually increase the observed scattering. A wide variety of results might thus be obtained under different experimental conditions.

REFERENCES

Aditya, P. K., 1959, *Indian J. Phys*, **33**, 357

——— 1964a, *Nuovo Cimento*, **31**, 473.

——— 1964b, *Indian J. Phys.*, **38**, 326.

——— Bhatia, V. S and Sood, P M, 1963, *Nuovo Cimento* **29**, 577

Aditya, P. K., and Puri, R. K., 1964, *J. Sci. Instrum* **41**, 529

Biswas, S., Peters, B., and Rama, 1955, *Proc. Ind Acad Sci.*, **41**, 154

Biswas, S., Durgaprasad, N., and Mitra, S., 1957, *Proc. Ind Acad. Sci*, **46**, 167.

Bozoki, G., Gombosi, E, Janik, L., Nagy, E, and Sahni, M., 1961, *Private communication* (unpublished)

Brisbout, F. A., Dahanayake, C, Engler, A, Fowler, P H., and Jones, P. B., 1956, *Nuovo Cimento*, **2**, 1400.

Fowler, P. H., 1950, *Phil Mag*, **41**, 169.

Hossain, A., Votruba, M F., and Wataghin, A., 1961, *Nuovo Cimento*, **22**, 308.

Kamal, A A., Rao, G K., and Rao, Y V, 1964, *Nuovo Cimento*, **32**, 863.

Menon, M G K, O'Ceallaigh, C, and Rochat, O., 1951, *Phil. Mag*, **42**, 932.

Pal, Y, and Ray, A. K, 1963, *Nuovo Cimento*, **27**, 960.

Voyvodic, L, and Pickup, E., 1952, *Phys Rev*, **85**, 91